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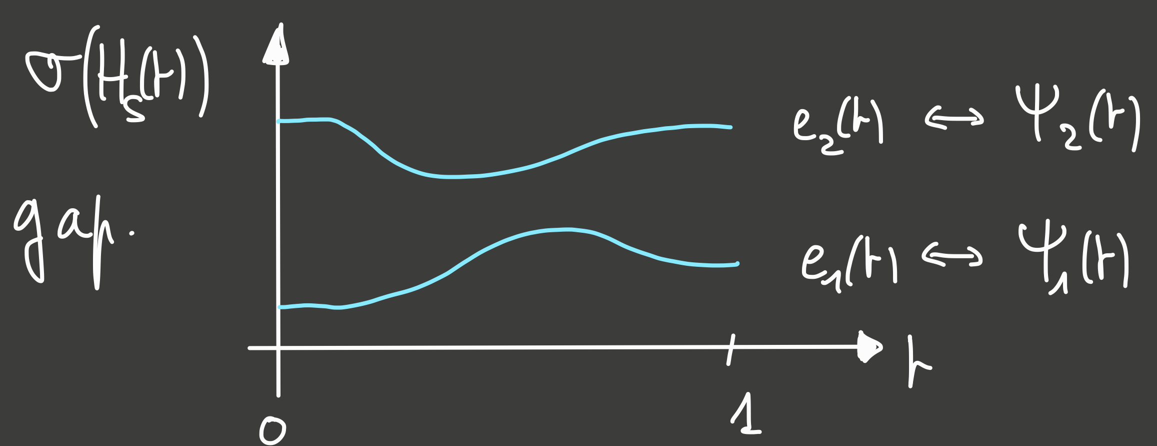
- System: On $\mathbb{C}^2$;

$$H_S(t) = H_S^*(t) = \sum_j e_j(t) P_j(t)$$



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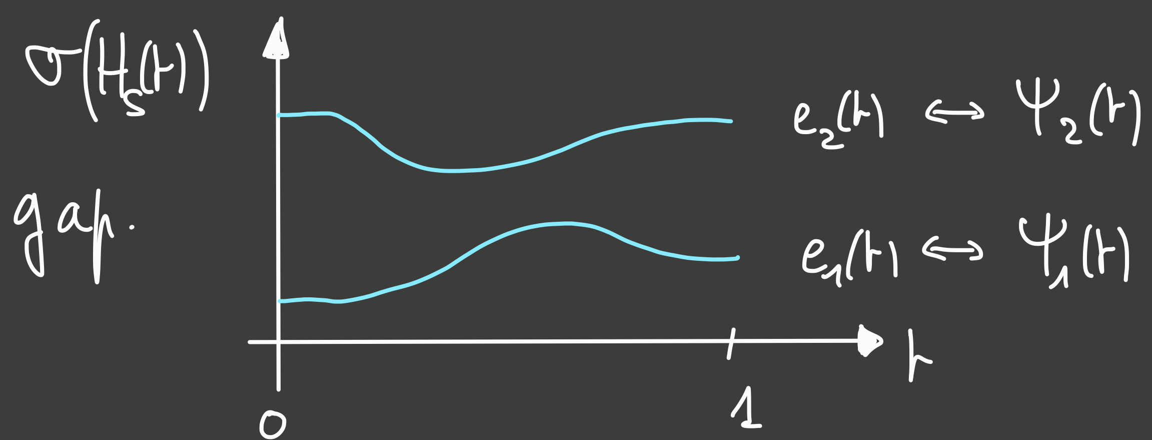

$$\begin{cases} i\varepsilon \partial_t \Psi_S(t) = H_S(t)\Psi_S(t) & ; \quad t \in (0,1) \\ \Psi_S(0) \in \mathbb{C}^2, \text{ normalized}, & \varepsilon \to 0. \end{cases}$$

- Environment: $\mathcal{F}_+(L^2(\mathbb{R}^3))$ Fock space, vacuum vector χ.

  $a^*(f)$, $a(f)$ creators/annihilators, $f \in L^2(\mathbb{R}^3)$; CCR: $[a(f), a^*(g)] = \langle f | g \rangle_{L^2} \mathbb{1}$

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Field op: $\Phi(g) = \frac{1}{\sqrt{2}}\left(a^*(g) + a(g)\right)$; g form factor

$k \mapsto \omega(k) = |k|$ dispersion relation and $H_R = \int_{\mathbb{R}^3} dk^3 \, \omega(k) \, a^*(k) \, a(k)$ field Hamiltonian

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- Coupling: on $\mathcal{H} := \mathbb{C}^2 \otimes \mathcal{F}_+(L^2(\mathbb{R}^3))$, Hilbert space of the full system "2-level plus field"

$H_{int}(t) = \lambda\, B(t) \otimes \phi(g)$; $B(t) = B^*(t)$ s.t. $[B(t), H_S(t)] \equiv 0$.

i.e. "Energy conserving" instantaneously

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- Evolution operator: on $\mathcal{H}$, $H_{tot}(t)$

$$\begin{cases} i\varepsilon \partial_t U(t) = \left(H_S(t) \otimes \mathbb{1} + \lambda B(t) \otimes \phi(g) + \mathbb{1} \otimes H_R \right) U(t) ; \\ U(0) = \mathbb{1} ; \quad t \in (0,1) \end{cases}$$

$\leftarrow$ (ε, λ) dependent unitary.

as $(\varepsilon, \lambda) \to 0$.

vacuum.

- State: $\rho(0) = \rho_1(0) \otimes |\chi\rangle\langle\chi|$ $\rightsquigarrow$ Reduced state $\rho_S(t) = \mathrm{tr}_R\left[U(t)\, \rho(0)\, U^*(t) \right]$ on $\mathbb{C}^2$

Rem: For time indep. Hamiltonian: $[B, H_S] = 0 \Rightarrow \left(\rho_S(t)\right)_{jj} \equiv \left(\rho_S(0)\right)_{jj} \quad \left(= \delta_{j1} \right).$

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- **Adiabatic transition probability:**

indep. of field state

vacuum

$$p^{(\varepsilon,\lambda)}_{1\to 2}(t) := \mathrm{tr}_{\mathbb{C}^2}\left(P_2(t)\, \rho_S(t) \right) = \mathrm{tr}\left(\left(P_2(t) \otimes \mathbb{1} \right) U(t) \left(P_1(0) \otimes |\chi\rangle\langle\chi| \right) U^*(t) \right) \text{ as } (\varepsilon,\lambda) \to 0.$$

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- In absence of field (2-level syst.), assuming $H_S^{(n)}(0) = 0$, $n = 1, 2, 3 \ldots$

Born-Fock '28, Kato '50, ...

$$q_{1\to2}(t) = \frac{\left|\langle \psi_2(t) | P_1'(t) \psi_1(t)\rangle\right|^2}{\left(e_1(t) - e_2(t)\right)^2} \quad \left(= q_{2\to1}(t)\right).$$

$$p^{(\varepsilon,0)}_{1\to2}(t) = \varepsilon^2 q_{1\to2}(t) + O(\varepsilon^3);$$

$\psi_j(t)$ (smooth) instantaneous eigenvectors.

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- Adiabatic transition probability :

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$$p^{(\varepsilon,\lambda)}_{1\to 2}(t) := \mathrm{tr}_{\mathbb{C}^2}\left(P_2(t)\, \rho_S(t)\right) = \mathrm{tr}\left(\left(P_2(t) \otimes \mathbb{1}\right) U(t) \left(P_1(0) \otimes |\Omega\rangle\langle\Omega|\right) U^*(t)\right) \text{ as } (\varepsilon,\lambda) \to 0.$$

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- Goal : compare $p^{(\varepsilon,\lambda)}_{1\to 2}$ and $p^{(\varepsilon,0)}_{1\to 2}(t)$ for $(\varepsilon,\lambda) \to 0$; $\lambda \neq 0$.

- Reservoir auto-correlation funct: $\omega(k) = |k|$; $g(k) = g(\omega, \sigma)$ sph. coord.

$$\gamma(t) := 2\langle X | e^{itH_R} \phi(g) e^{-itH_R} \phi(g) X \rangle = \langle e^{it\omega} g | g \rangle_{L^2} = \int_0^\infty e^{-i\omega t} \underbrace{\left\{ \omega^2 \int_{S^2} d\sigma^2 \, | g(\omega, \sigma)|^2 \right\}}_{:= \frac{1}{2\pi} \hat{\gamma}(\omega) \mathbb{1}_{\{\omega \geq 0\}}} d\omega$$

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We assume $\int_{\mathbb{R}} |\gamma(t)| dt + \int_{\mathbb{R}} |t \, \gamma(t)| dt < \infty$. (for simplicity)

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- Thm:

$\sim \varepsilon^2$ $\qquad$ $\sim \lambda^2 \varepsilon$ $\qquad$ $\begin{cases} = 0 & \text{if } e_1 \leq e_2 \\ > 0 & \text{if } e_1 > e_2 \end{cases}$

$$p^{(\varepsilon,\lambda)}_{1\to 2}(t) = p^{(\varepsilon,0)}_{1\to 2}(t) + \frac{\lambda^2}{2\varepsilon} \int_0^t p^{(\varepsilon,0)}_{1\to 2}(s)\, (\ell_1 - \ell_2)^2(s)\, \hat{\gamma}\big((e_1 - e_2)(s)\big)\, ds + \text{error}(\varepsilon, \lambda)$$

where: $\varepsilon \ll \lambda \ll \varepsilon^{1/3}$ ensures $\text{error}(\varepsilon,\lambda) \ll \min(\varepsilon^2, \lambda^2\varepsilon)$; $\lambda = \sqrt{\varepsilon}$ same order.

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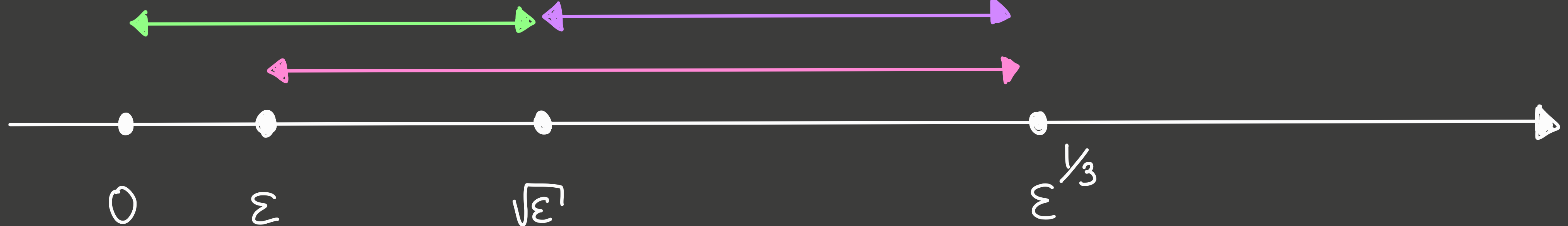

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Avron - Fraas - Graf - Grech ('11, '12) and Fraas - Hänggli ('17)

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<u>Rem:</u> Time indep. case $\Rightarrow$ conserves functions of H^S

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$$H^S_t = \frac{1}{2}\begin{pmatrix} -t & \delta \\ \delta & t \end{pmatrix} \quad ; \quad D_t(\rho) = -\left(P_1(t)\rho P_2(t) + P_2(t)\rho P_1(t)\right)$$

$$\Rightarrow p^{(\varepsilon,\gamma)}_{1\to 2}(t) = \gamma\varepsilon \int_0^t q_{1\to 2}(s)\, ds + O(\varepsilon^2) + O(\varepsilon\gamma^3)$$

"similar" with $\gamma = \lambda^2$

Non zero temperature : $T = 1/\beta > 0$.

Eq. momentum distribution via Planck's law : $\frac{1}{e^{\beta \omega(k)} - 1}$; $\omega(k) = |k|$,

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$\rho_{1\to 2}^{(\lambda,\varepsilon)}(t) = \mathrm{tr}_2\Big(\big(P_2(t)\otimes \mathbb{1}\big)\, U(t)\, \big(P_1(0)\otimes \rho_{R,\beta}\big)\, U^*(t)\Big)$ as $(\varepsilon,\lambda)\longrightarrow 0$; for fixed $\beta > 0$.

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The auto-correlation function is now:

T = 0 expression

$$\hat{\gamma}^{\beta}(\omega) = \hat{\gamma}(|\omega|)\left(\coth(\beta|\omega|/2) + \mathrm{sgn}(\omega)\right)/2 \geq 0 \quad \& \quad \gamma^{\beta}(t) := \frac{1}{2\pi}\int_{\mathbb{R}} e^{-i\omega t}\hat{\gamma}^{\beta}(\omega)\, d\omega.$$

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Same expression holds with $\hat{\gamma} \mapsto \hat{\gamma}^{\beta}$:

- Thm:

$\sim \varepsilon^2$ $\qquad \sim \lambda^2 \varepsilon$ $\qquad$ contributes if $e_1 < e_2$ and $e_2 < e_1$.

$$p^{(\varepsilon,\lambda)}_{1\to 2}(t) = p^{(\varepsilon,0)}_{1\to 2}(t) + \frac{\lambda^2}{2\varepsilon} \int_0^t p^{(\varepsilon,0)}_{1\to 2}(s) \, (h_1 - h_2)^2(s) \, \hat{\gamma}^{\beta}\big((e_1 - e_2)(s)\big)\, ds + \mathrm{error}_{\beta}(\varepsilon, \lambda)$$

On behalf of all participants, here & there, many thanks again to

Caroline

Clotilde

Rémi

for making this event possible!