DESINGULARIZATION OF CLOSED SEMIALGEBRAIC SETS AND APPLICATIONS

ANTONIO CARBONE (JOINT WORK WITH JOSÉ F. FERNANDO)

Bierstone and Parusiński studied the desingularization of d-dimensional closed subanalytic sets and in particular of d-dimensional closed semialgebraic sets. Their procedure preserves the number of d-dimensional components connected by analytic paths of the involved closed semialgebraic sets, so they have a good behaviour for pure dimensional closed semialgebraic sets. If the involved d-dimensional closed semialgebraic set is not pure dimensional, some components connected by analytic paths of smaller dimension could be dropped during the desingularization process. For instance, Whitney's umbrella $W := \{\mathtt{y}^2\mathtt{z} - \mathtt{x}^2 = 0\} \subset \mathbb{R}^3$ has two components connected by analytic paths (one of dimension 2 and the other of dimension 1), whereas its desingularization has only one component connected by analytic paths, that has dimension 2. The obtained models in the desingularization process, that we call *closed chessboard sets*, are the closures of (finite) unions of connected components of the complements of normal-crossings divisors of non-singular real algebraic sets.

In this seminar we start by presenting the resolution of d-dimensional closed chessboard sets $\mathcal{S}$ using Nash manifolds with corners $\mathcal{Q}$ with the same number of connected components as $\mathcal{S}$ (or equivalently the same number of irreducible components). Next, we present the unfolding $U(\mathcal{Q})$ of a Nash manifold with corners $\mathcal{Q}$, which is the analogous of the Nash double of a Nash manifold with smooth boundary, but takes into account the peculiarities of the boundary of $\mathcal{Q}$. Combining the previous results with Bierstone and Parusiński's desingularization we obtain that a closed semialgebraic set $\mathcal{S}$ connected by analytic patths can be desingularizad by a Nash manifolds with corners $\mathcal{Q}$ with the same number of connected components. Moreover, $\pi : U(\mathcal{Q}) \to \mathcal{S}$ is a Nash ramified covering of constant finite degree.

Finally, we present several applications of our results:

- A complete characterisation of the compact semialgebraic sets that are images of the unitary closed ball under Nash maps.
- A compactification result for Nash manifolds with corners by compact Nash manifolds with corners.
- A result on Nash approximation of continuous semialgebraic maps whose target space are Nash manifolds with corners.