MAT137Y1 – LEC0501

Calculus!

Trigonometric and rational functions

January 30th, 2019

For next week

For Monday (Feb 4), watch the videos:

- Volumes: 10.1

For Wednesday (Feb 6), watch the videos:

- Volumes: 10.2
- Sequences: 11.1, 11.2

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

Hint: You will need

- $\frac{d}{dx}[\tan x] = \ldots$
- $\frac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

Practice: Integrals with trigonometric functions

Compute the following antiderivatives.
Once you get them to a form from where it is easy to finish, you may stop: we are only interested in the method!

1. $\int \sin^{10} x \cos x \, dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int e^{\cos x} \cos x \sin^5 x \, dx$
4. $\int \cos^2 x \, dx$
5. $\int \sin^4 x \, dx$
6. $\int \csc x \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Rational integrals

1. Compute $\int \frac{1}{x+a}\,dx$
2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x+3}$
3. Compute $\int \frac{-x+6}{x^2+3x}\,dx$
4. Compute $\int \frac{1}{x^2+3x}\,dx$
5. Compute $\int \frac{1}{x^3-x}\,dx$

Repeated factors

1. Compute $\int \frac{1}{(x+1)^n}\,dx$ for $n > 1$
2. Compute $\int \frac{x}{(x+1)^2}\,dx$
3. Compute $\int \frac{3x+2}{(x+1)^2}\,dx$
4. Compute $\int \frac{x^3}{(x+1)^2}\,dx$

Irreducible quadratics

1. Compute $\int \frac{1}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$.
2. Compute $\int \frac{2x+3}{x^2+1}\,dx$
3. Compute $\int \frac{x^2}{x^2+1}\,dx$
4. Compute $\int \frac{1}{x^2+x+1}\,dx$

A reduction formula – *Homework*

Let $I_n = \int_0^{2\pi} \sin^n x\,dx$.

1. Compute I_0 and I_1.
2. Starting with I_n, use integration by parts. Then use the main trig identity to obtain an equation involving I_n and I_{n-2}.
3. Compute I_8 and I_{55}.