Monotonicity

UNIVERSITY OF TORONTO

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For next lecture

For Wednesday (Nov 28), watch the videos:

- Indeterminate forms and L'Hôpital's Rule: (6.3), 6.4, 6.5, (6.6), 6.7, 6.8, (6.9), 6.10

Inequalities

Prove that, for every $x \in \mathbb{R}$,

$$e^x \geq 1 + x$$

Hint: study the monotonicity of $f(x) = e^x - 1 - x$.

Intervals of monotonicity

Let $g(x) = x^3(x^2 - 11)^{1/3}$.

1. What is the domain of definition of g?
2. What is the domain of differentiability of g?
3. Study the monotonicity of g.
4. Sketch the graph of g.

To save time, here is the first derivative:

$$g'(x) = \frac{11x^2(x^2 - 9)}{3(x^2 - 11)^{2/3}}$$

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Monotonicity and extrema

Study the extrema of

$$f(x) = \frac{x^2}{x^2+1}$$

$$g(x) = \frac{x^2+x+1}{x^2+1}$$