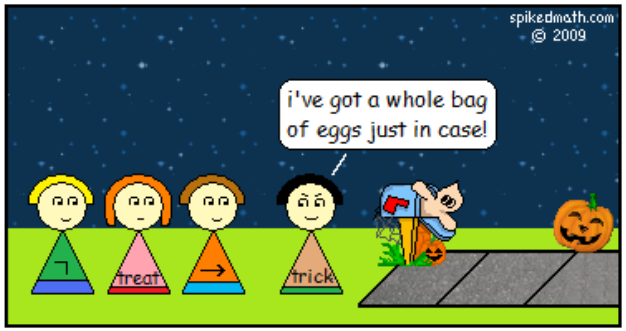
Related rates & derivatives of exp and log

UNIVERSITY OF TORONTO



October 31st, 2018

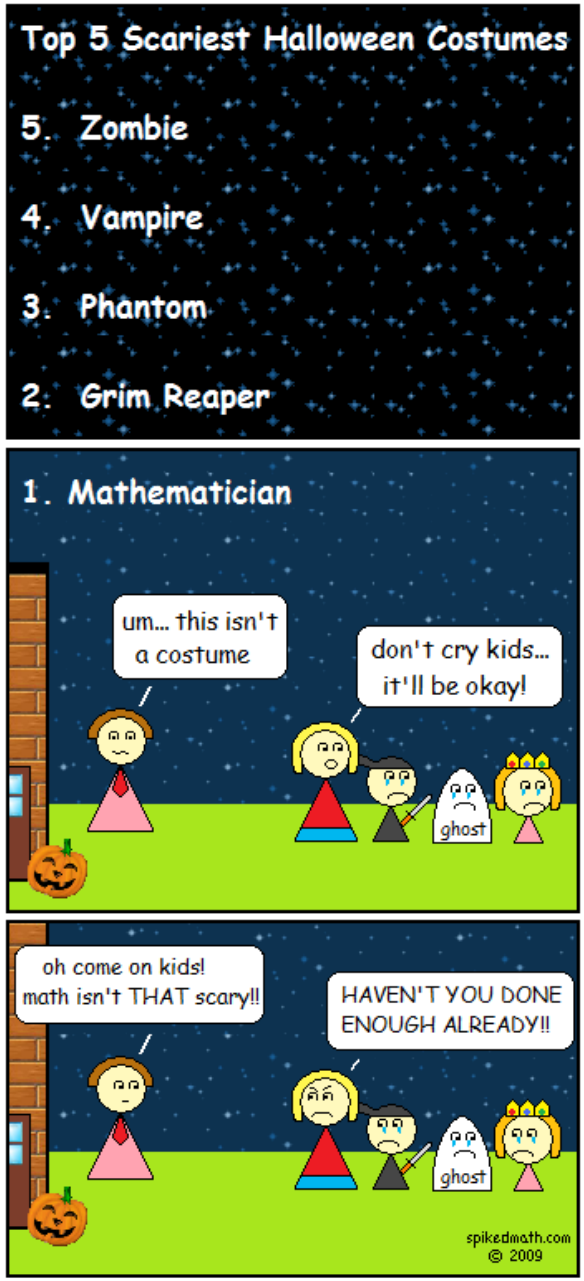
For (next)×2 week

For Monday (Nov 12), watch the videos:

- Inverse functions: 4.1, 4.2

For Wednesday (Nov 14), watch the videos:

- One-to-one functions: 4.3, 4.4, 4.5
- Inverse trig functions: 4.6, 4.7, 4.8



Estimations

1. We know

$$f(0) = 2, \quad f'(0) = 3, \quad g(0) = 7, \quad g'(0) = 6.$$

Compute $\lim_{x \to 0} \frac{f(x)}{g(x)}$.

2. We know

$$f(0) = 0, \quad f'(0) = 3, \quad g(0) = 0, \quad g'(0) = 6.$$

When x is close to 0, give estimates for $f(x)$ and $g(x)$.

Then compute $\lim_{x \to 0} \frac{f(x)}{g(x)}$.

Warm up

Compute the derivative of the following functions using that $\frac{d}{dx}e^x = e^x$ and $\frac{d}{dx}\ln(x) = \frac{1}{x}$.

1. $f(x) = a^x$
2. $f(x) = \log_a(x)$
3. $f(x) = e^{\sin x + \cos x} \ln x$
4. $f(x) = \pi^{\tan x} + \sqrt{e^x}$
5. $f(x) = \ln\left(e^x + \ln(\ln(\ln(x)))\right)$
6. $f(x) = e^{\sin(x)+x^3}$
7. $f(x) = \frac{\sin\left(e^x+x^2\right)}{x^2+1}$

Computations!

1. Compute the derivative of

$$f(x) = \log_{x+1}(x^2 + 1)$$

2. Compute the derivative of

$$g(x) = x^{\tan x}.$$

3. Compute the derivative of

$$h(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

Sleepy ants

Two ants are taking a nap. The first one is resting at the tip of the minute hand of a cuckoo clock, which is 25 cm long. The second one is resting at the tip of the hour hand, which is half the length. At what rate is the distance between the two ants changing at 3:30?

Make this function differentiable

Find $a, b \in \mathbb{R}$ such that the following function is differentiable on $(0, +\infty)$:

$$f(x) = \begin{cases} \sqrt{x} & \text{if } 0 \leq x \leq 1 \\ ax^2 + bx + 1 & \text{if } x > 1 \end{cases}$$

Computations![1]

Compute the derivatives of the following functions:

1. $f(x) = \tan(3x^2 + 1)$
2. $f(x) = x(\sin 2x)(\tan 3x)$
3. $f(x) = \cos(\sin(\tan x))$
4. $f(x) = \cos\left(3x + \sqrt{1 + \sin^2 x^2}\right)$

[1] This slide was not used during the class. You can use it to train yourself.

Lake ripple[2]

We drop a pebble into a lake. It produces a circular ripple. When the radius is 2 meters and is increasing at a rate of $10cm/s$, at what rate is the area increasing?

[2] This slide was not used during the class. You can use it to train yourself.

A discontinuous function!