

Théories asymptotiques et équations de Painlevé

1-5 juin 2004 — ANGERS (France)

Résumé des conférences / Abstracts

Philip BOALCH

Title: Painlevé equations and complex reflections

Abstract: The aim of this talk is to explain the relation between three-dimensional complex reflection groups and the sixth Painlevé equation. For example, this relation (and a result of Jimbo) allows us to construct an explicit algebraic solution, the "Klein solution", starting from the Klein complex reflection group of order 336. It turns out that this solution is not equivalent to any solution coming from a finite subgroup of $SL_2(C)$.

Alexei BORODIN

Title: Isomonodromy transformations of difference equations and discrete Painlevé equations.

Abstract: The goal of the talk is to explain how the difference and q -Painlevé equations arise in the context of isomonodromy transformations of linear systems of (q -)difference equations with rational coefficients. The isomonodromy interpretation leads to applications of the discrete Painlevé equations in certain probabilistic models of representation theory, combinatorics, and percolation.

Peter CLARKSON

Title: Rational solutions and associated special polynomials for the Painlevé equations.

Abstract: In this talk I shall discuss rational solutions and associated polynomials for the second, third, fourth and fifth Painlevé equations (P_{II} – P_V). The Painlevé equations are six nonlinear ordinary differential equations that have been the subject of much interest in the past twenty-five years, which have arisen in a variety of physical applications and may be thought of as nonlinear special functions. Rational solutions of the Painlevé equations are expressible as the logarithmic derivative of special polynomials. For P_{II} these special polynomials are known as the *Yablonskii-Vorob'ev polynomials*. The locations of the roots of these polynomials is shown to have a highly regular triangular structure in the complex plane. The analogous special polynomials for P_{III} , P_{IV} and P_V are derived and I shall show that the roots of these special polynomials also have a highly regular structure.

Robert CONTE

Title: Completeness of the cubic and quartic Henon-Heiles Hamiltonians.

Abstract: The autonomous Hénon-Heiles Hamiltonian with additional inverse square terms passes the Painlevé test for only seven values of the parameters (three in the cubic case, four in the quartic case), which are all integrable in the Liouville sense. Four of them have been integrated by hyperelliptic functions of genus two. We integrate the three remaining cases by establishing a birational transformation with three differential equations in the classification (Cosgrove, 2000) of fourth order first degree equations in a special polynomial class which possess the Painlevé property. This also proves that the autonomous Hénon-Heiles Hamiltonian is complete, in the usual Painlevé sense.

Ovidiu COSTIN

Title: Discrete analog of the Painlevé property and integrability of difference equations.

Abstract: Techniques of Écalle transseries and generalized Borel summability are used to find unique continuation in the complex plane of solutions of a relatively general class of difference equations. This continuation allows for Painlevé property methods to be extended to difference equations. In higher order equations new types of transseries-like objects emerge naturally.

It is shown that the Painlevé property (PP) induces, under relatively general assumptions, a dichotomy within first order difference equations: all equations with PP can be solved in closed form; on the contrary, absence of PP implies, under some further assumptions, that the local conserved quantities are strictly local in the sense that they develop singularity barriers on the boundary of some compact set. Higher order equations will be discussed as well.

Based on work in collaboration with Martin Kruskal,
to appear in Comm. Pure Appl. Math.

Philippe Di FRANCESCO

Title: Geodesic distance in planar graphs: exact results via solitons.

Abstract: Using a bijection between various classes of planar graphs and suitably decorated trees, we compute the generating function for graphs with two marked points at a fixed geodesic distance. The results are obtained as KP-soliton-like solutions of discrete Painlevé-like equations, and give access to the fractal dimension of the underlying surfaces. We also display another bijection between planar graphs and labeled trees, which allows to view planar graphs as spatially branching processes in one dimension, and to interpret elliptic solutions to our equations as escape probabilities of spreading species from some interval.

Boris DUBROVIN

Title: Riemann-Hilbert problem and quantum canonical transformations.

Abstract: I will discuss a certain class of canonical transformations acting on the Givental symplectic space of functions on the unit circle with values in a linear space equipped with a metric. The transformations are defined in terms of matrix valued functions $G(z)$ on the unit circle satisfying $G^*(-z)G(z) = 1$. It will be explained, following my joint work with Y.Zhang, how to apply Riemann-Hilbert problem to quantizing these linear canonical transformations. Applications to Gromov-Witten invariants of the projective line and to enumeration of ribbon graphs/triangulations will also be discussed.

Davide GUZZETTI

Title: The elliptic representation of the 6th Painlevé equation.

Abstract: I will present a technique to study the general 6th Painlevé equation based on the elliptic representation in combination with the isomonodromy deformation theory. The technique allows to obtain the critical behavior of almost all solutions and to solve the connection problem.

Alexander ITS

Title: Painlevé Transcendents and Random matrices.

Abstract: Painlevé functions have been playing increasingly important role in random matrix theory since the early nineties works on the 2d quantum gravity of Brézin and Kazakov, Douglas and Shenker, and Gross and Migdal and the works (also early nineties) of Mehta and Mahoux, and Tracy and Widom on the level spacing distributions. Indeed, the appearance of Painlevé functions in the field can be traced to 1970s-1980s works of Barouch, McCoy, Tracy, and Wu, and of Jimbo, Miwa, Mori and Sato on the quantum correlation functions. In this talk we will try to review these, and also some of the more recent results concerning Painlevé transcendents and random matrices. We will present a unified point view on the subject based on the Riemann-Hilbert method. The emphasis will be made on the various double scaling limits related to the universal properties of random matrices for which Painlevé functions provide an adequate “special function environment”.

Katsunori IWASAKI

Title: Dynamics of the sixth Painlevé equation.

Abstract: I shall try to give a total picture of the sixth Painlevé equation from geometrical and dynamical points of view. It involves such keywords as symplectic geometry, moduli of stable parabolic connections, moduli of monodromy representations, Riemann-Hilbert correspondence, geometry of cubic surfaces, braid and modular groups, rational double points and their resolutions of singularities, affine Weyl groups, and discrete dynamical systems. This talk is based on a joint work with M. Inaba and M.-H. Saito.

Michio JIMBO

Title: Form factors and representation theory of quantum affine algebras

Abstract: In integrable quantum field theory in two dimensions, local fields are described by a series of meromorphic functions called form factors. One of the basic issues is to classify all local fields in the theory and to establish a one-to-one correspondence with those in conformal field theory. We give a review on some recent progress based on representation theory of the quantum affine algebra $U_q(\widehat{sl}_2)$ at $q = \sqrt{-1}$.

Nalini JOSHI

Title: Hunting Nonlinear Mathematical Butterflies.

Abstract: Nonlinear partial and ordinary differential equations that arise in mathematical physics generically have highly transcendental solutions. But some of these solutions have very simple asymptotic series expansions, unstable in solution space like the Lorenz butterfly of chaos theory, in certain limits. The trouble is that these series are typically divergent and it is very difficult to extract any information about the solutions from them. I will review the difficulties for the Painlevé equations and their hierarchies and some techniques for solving them.

Alexander KITAEV

Title: Selected Topics in Asymptotic Mosaic for Painlevé Equations

Abstract: In the first part of the talk we review asymptotic results for the first and second Painlevé equations with the main perspective on Nevanlinna theory. The second part is devoted to RS -transformations and their application to asymptotics of the Painlevé equation.

Bernard MALGRANGE

Titre : Déformations isomonodromiques et fonction τ .

Résumé : On s'intéresse aux relations entre la structure hamiltonnienne des équations de Schlesinger d'une part, la propriété de Painlevé d'autre part. Les relations entre ces deux propriétés, et notamment l'intervention de la même fonction τ , sont interprétées à partir de la forme de Liouville de (l'extension centrale de) l'espace des lacets.

Marta MAZZOCCO

Title: On the second Painlevé hierarchy.

Abstract: The second Painlevé hierarchy will be introduced. We will discuss special solutions, asymptotic behaviour and canonical transformations.

Pierre van MOERBEKE

Titre : Distributions statistiques en combinatoire, modèles matriciels et équations de Painlevé.

Résumé : Les questions suivantes :

- (1) La distribution statistique du spectre de matrices aléatoires,
- (2) la distribution de la longueur de la plus longue sous-suite croissante de permutations aléatoires et
- (3) la distribution de marches aléatoires sur les entiers sans se couper,

constituent trois problèmes étroitement liés aux solutions des équations de Painlevé.

Ces questions évoquent de nombreuses questions asymptotiques: quelle distribution trouve-t-on, lorsque la taille des matrices aléatoires tend vers l'infini? Quelle est la distribution de la longueur de la plus longue sous-suite croissante lorsque la longueur des permutations tend vers l'infini? Quelle est la distribution du marcheur extérieur, lorsque le nombre de marcheurs augmente.

Toutes ces distributions statistiques sont solutions d'équations de Painlevé; l'asymptotique des équations de Painlevé joue un rôle important dans ces résultats. Ils sont intimement liés à des modèles matriciels et les systèmes intégrables, comme les équations de Korteweg-de Vries, les réseaux de Toda, Pfaff et Toeplitz.

Juan MORALES RUIZ

Titre : A remark about the Painlevé transcendents

Abstract: The Painlevé transcendents are Hamiltonian systems that, except for P_I , depend on parameters. Moreover, for some values of the parameters they have rational or transcendental particular solutions. So, in these cases, we can apply the Galoisian method to the variational equation along these particular solutions in order to obtain a non-integrability criteria: if the identity component of the Galois group of the variational equation is non-abelian, the Hamiltonian system is not completely integrable, with meromorphic or rational first integrals. The idea of this talk comes from some discussions with J.-A. Weil.

Masatoshi NOUMI

Title: Elliptic difference Painlevé equation and its hypergeometric solutions.

Abstract: The elliptic Painlevé equation is a second order difference equation with affine Weyl group symmetry of type E_8 . This equation is defined by the Cremona transformations on \mathbf{P}^2 relative to the configuration of 9 points in general position (Sakai, 2001), and is thought of as the master equation for all discrete Painlevé equations of second order. In this talk I will give an explicit description of the elliptic Painlevé equation and introduce a framework of tau functions for it. Also I will discuss its Riccati solutions along a reflection hyperplane in the parameter space; they are expressed by elliptic hypergeometric functions. This talk is based on a joint work with K.Kajiwara, T.Masuda, Y.Ohta and Y.Yamada.

Kazuo OKAMOTO

Title: The Painlevé systems and the Garnier systems.

Abstract: The aim of the lecture is double: we look firstly over basic facts on Painlevé systems such as, Hamiltonian structure, Bäcklund transformations, bilinear forms and so on associated to the system; the second, and very important, part of the lecture concerns recent developments of researches on Painlevé systems and on Garnier systems. In particular, we deal with bilinear forms of the Painlevé systems. Many results on this subject are known and, among them, we give the whole list of the bilinear forms which are reduced from the non-linear differential equations satisfied by Hamiltonian functions. Moreover, studies on the Painlevé systems as well as the Garnier systems are related to many other fields of mathematics and mathematical physics. We investigate them from various points of view by means of analytic, algebraic and geometric method as introduced in the beginning of the lecture. This point causes one of the most important reasons why researches on Painlevé systems interest us and why they are so fruitful.

Alfred RAMANI

Title: Discrete Painlevé equations.

Abstract: We identify, using various criteria developed over the past decade, a large number of nonlinear second-order recursion relations which share many properties with the (continuous) Painlevé equations. Moreover these discrete systems go over to the latter at the continuous limit. We shall show that these mappings can legitimately be considered as the discrete analogues of the Painlevé equations.

Jacques SAULOY

Title: Isomonodromy for q -difference equations.

Abstract : We interpret the isomonodromy condition for q -difference equations as given in the seminal paper of Jimbo and Sakai in terms of existence of a vector bundle over the product of two elliptic curves. We study the special case of fuchsian families.

Yoshitsugu TAKEI

Title: Hierarchies of Painlevé equations and exact WKB analysis.

Abstract : We discuss several hierarchies of (higher-order) Painlevé equations, particularly their Stokes geometries, from the viewpoint of exact WKB analysis.

Hiroshi UMEMURA

Title: Some questions related with the Painlevé equations.

Abstract: Since the discovery of the Painlevé transcendents a hundred years ago, we do not cease to know their nature of special functions. Combinatorial characters revealed recent years are one of the important instances. We discuss some topics about combinatorics and Painlevé equations, and differential Galois theory.

Changgui ZHANG

Titre : Sur une algèbre de fonctions asymptotiques q -Gevrey.

Résumé : La théorie analytique des équations aux q -différences linéaires a connu ces dernières années des progrès en matière de description (algébrique, géométrique, analytique) de la singularité. Pourtant des modèles non linéaires sont encore peu compris : en particulier, aucune méthode de sommation ne s'y applique sans modification notable. Dans l'exposé, on expliquera comment cette problématique nous amène à introduire une algèbre de fonctions qui sont toutes soumises à une double contrainte : asymptotique (version Gevrey q -analogue) et algébrique (pôles et ses multiplicités).