

Memichi

Eilenberg - Moore spectral sequence
and string topology

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18h40 - 19h15

(1)

joint work with my japanese colleagues

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I String topology of manifold

Let M be a closed oriented manifold of dimension m .

Denote by LM the space of free loops on M

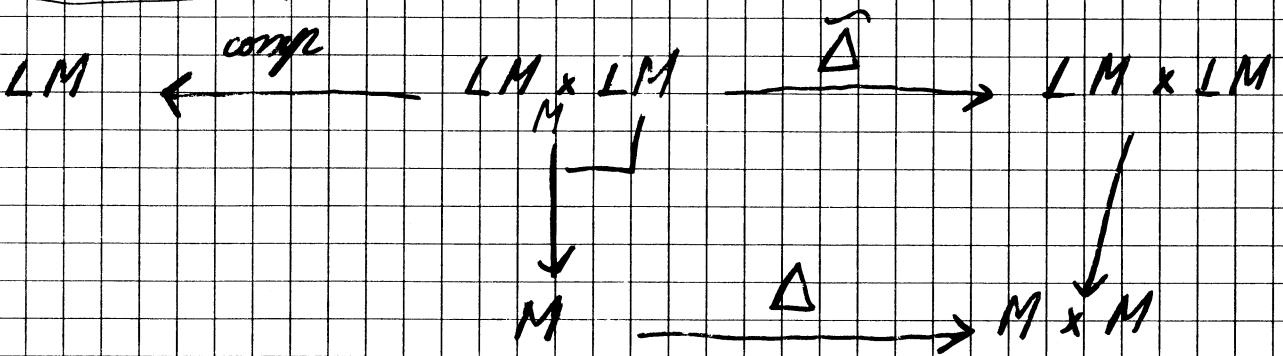
Chas and Sullivan have defined a product called

the loop product on the homology of free loops space of M

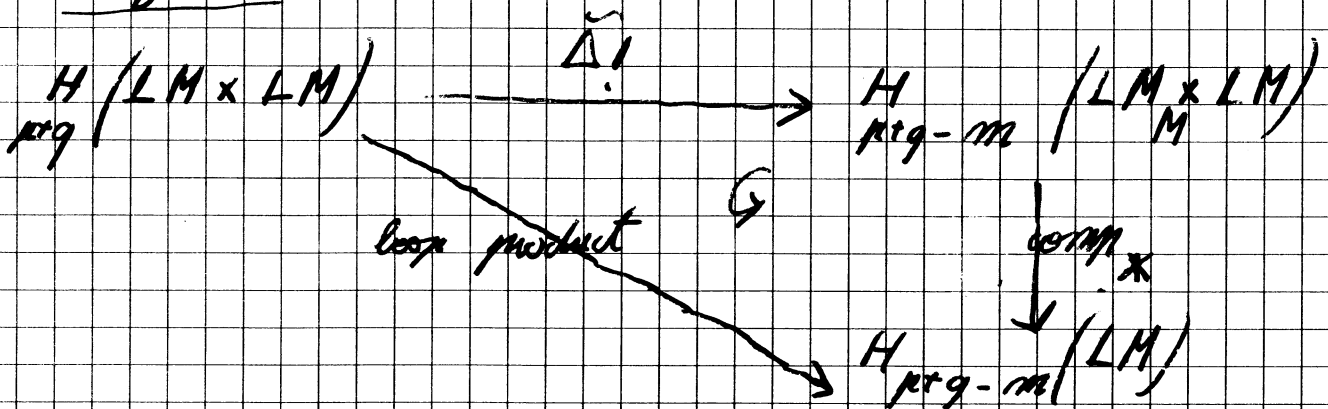
$$H_p(LM) \circ H_q(LM) \longrightarrow H_{p+q-m}(LM)$$

Cohen - Jones definition

Consider the pull-back



Using Thom - Pontryagin construction, Cohen and Jones defined



II Felix - Thomas extension to Gorenstein space

def An augmented differential graded algebra A
is a Gorenstein algebra of dimension $m \in \mathbb{Z}$

iff

$$\dim \text{Ext}_{A/\mathbb{F}, A}^l = \begin{cases} 0 & \text{if } l \neq m \\ 1 & \text{if } l = m \end{cases}$$

def A topological space M is a Gorenstein space
if the singular cochain algebra $C^*(M)$ is a
Gorenstein algebra

- Examples 1) closed oriented manifold M $m = \dim M > 0$
2) classifying space of connected Lie group BG
 $m = -\dim G < 0$

Ex. F-T 1) Let M be a \mathbb{Z} -connected Gorenstein
space of dimension m .

Then

$$\text{Ext}_{C^*(M^2)}^* (C^*(M), C^*(M^2)) \cong \mathbb{F} H^*(X)^{*-m}$$

Let $\Delta^!$ denote a generator of $\text{Ext}_{C^*(M^2)}^m (C^*(M), C^*(M^2))$
corresponding to $1 \in \mathbb{F} H^0(M)$.
1) These elements are unique

$$\Delta^! \in \text{Ext}^m \left(C^*/(LM \times LM)_M, C^*/(LM \times LM) \right) \quad (3)$$

such that in the derived category of $C^*(M)$ -modules

the following diagram commutes

$$\begin{array}{ccc}
 C^*/(LM \times LM)_M & \xrightarrow{\Delta^!} & C^*/(LM \times LM) \\
 \uparrow \omega^* & & \uparrow (\omega \times \omega)^* \\
 C^*(M) & \xrightarrow{\Delta^!} & C^*(M \times M)
 \end{array}$$

The dual of the loop product is given by the composite

$$C^*(M) \xrightarrow{\text{loop}^*} C^*/(LM \times LM)_M \xrightarrow{\Delta^!} C^*/(LM \times LM)$$

III Rational isomorphism with Hochschild cohomology

Let M be a simply-connected Gorenstein space

Denote by $A(M)$ the algebra of polynomial differential forms on M introduced by Sullivan

$A(M)$ commutative dga $\simeq C^*(M; \mathbb{Q})$

from (KMN)

The Eilenberg-Moore isomorphism

$H_p(LM; \mathbb{Q}) \cong HH^{-p}(A(M), A(M)^\vee)$

Hochschild cohomology of $A(M)$ with coefficients in its dual