ERRATUM TO : BATALIN-VILKOVISKY ALGEBRAS AND CYCLIC COHOMOLOGY OF HOPF ALGEBRAS

LUC MENICHI

1. Introduction

There is a sign mistake in [4] and thefefore also in [5]. In the definition of Gerstenhaber algebra given in [4, 5], we gave the following Poisson rule:

"This means that for any $c \in G^k$ the adjunction map $\{-, c\} : G^i \to G^{i+k-1}, \quad a \mapsto \{a, c\}$ is a $(k-1)$-derivation: ie. for a, b, $c \in G$,

$$\{ab, c\} = \{a, c\}b + (-1)^{|a|(|c|-1)}a\{b, c\}. \tag{1}$$

"

This Poisson rule must be replaced by

"This means that for any $a \in G^k$ the adjunction map $\{a, -\} : G^i \to G^{i+k-1}, \quad b \mapsto \{a, b\}$ is a $(k-1)$-derivation: ie. for a, b, $c \in G$,

$$\{a, bc\} = \{a, b\}c + (-1)^{(|a|-1)|b|}b\{a, c\}. \tag{2}$$

"

The computation in section 9 of [5] must also be corrected: for example, in

"A tedious calculation shows that more generally in any Gerstenhaber algebra A, for $x_1, \ldots, x_p, y_1, \ldots, y_q \in A$,

$$\{x_1 \ldots x_p, y_1 \ldots y_q\} = \sum_{1\leq i\leq p, 1\leq j\leq q} \pm\{x_i, y_j\}x_1 \ldots \widehat{x_i} \ldots x_p y_1 \ldots \widehat{y_j} \ldots y_q.$$

where here $\pm$ is the sign $(-1)^{|x_i||x_1 \ldots x_{i-1}|+|y_j||y_1 \ldots y_{j-1}|+(|x_1 \ldots x_p|+1)|y_1 \ldots \widehat{y_j} \ldots y_q|}$."

This sign $\pm$ should be replaced by $(-1)^{|x_i||x_1 \ldots x_{i-1}|+|y_j||y_1 \ldots y_{j-1}|+(|y_j|-1)|x_1 \ldots \widehat{x_i} \ldots x_p|}$.

1991 *Mathematics Subject Classification.* 16W30, 19D55, 16E40, 18D50.

Key words and phrases. String Topology, Batalin-Vilkovisky algebra, Gerstenhaber algebra, Hochschild cohomology.

2. Explanation

The definition of Gerstenhaber algebra where the Lie bracket $\{-,-\}$ is a derivation with respect to the second variable (equation (2) above) is the classical one ([3, (9.14) and (9.18)], [2, Definition 1.1] or all my papers except [4, 5]).

But Gerstenhaber in his original paper [1] defined a Lie bracket in Hochschild cohomology which is a derivation with respect to the first variable (equation (1) above). That was our mistake in [4, 5].

Indeed, in [4, 5], our bracket $\{-,-\}$ (See [4, (2.7)] or [5, (7)]) is related to the Lie bracket $[-,-]$ of Gerstenhaber [1, (21) and (17) and Theorem 1] by

$$\{a,b\} = (-1)^{(|a|-1)(|b|-1)}[a,b] = -[b,a].$$

and it is easy to show the following Proposition:

Proposition 3. *Let G be a (upper) graded commutative algebra. Let*

$$[-,-] : G^i \otimes G^j \to G^{i+j-1}, \quad x \otimes y \mapsto [x,y]$$

and

$$\{-,-\} : G^i \otimes G^j \to G^{i+j-1}, \quad x \otimes y \mapsto \{x,y\}$$

be two linear maps of degree -1 related by

$$\{a,b\} = -[b,a].$$

for all a and $b \in G$.

If $[-,-]$ is a Lie bracket of degree -1 which is a derivation with respect to the first variable, explicitly, this means that for each a, b and $c \in G$

$[a,b] = -(-1)^{(|a|-1)(|b|-1)}[b,a]$,
$[a,[b,c]] = [[a,b],c] + (-1)^{(|a|-1)(|b|-1)}[b,[a,c]]$ *and*
$[ab,c] = [a,c]b + (-1)^{|a|(|c|-1)}a[b,c]$.
Then

$\{-,-\}$ is a Lie bracket of degree -1 which is a derivation with respect to the second variable, explicitly, this means that for each a, b and $c \in G$
$\{a,b\} = -(-1)^{(|a|-1)(|b|-1)}\{b,a\}$,
$\{a,\{b,c\}\} = \{\{a,b\},c\} + (-1)^{(|a|-1)(|b|-1)}\{b,\{a,c\}\}$ *and*
$\{a,bc\} = \{a,b\}c + (-1)^{(|a|+1)|b|}b\{a,c\}$.
And conversely.

This sign mistake was really annoying us. Because during the last ten years, we thought that the Hochschild cohomology of a symmetric (Frobenius) algebra was a Batalin-Vilkovisky algebra only in the sense of [4, 5] and not in the usual sense ([2, Proposition 1.2] or [3, paragrap 5.1]).

References

[1] M. Gerstenhaber, *The cohomology structure of an associative ring*, Ann. of Math. **78** (1963), no. 2, 267–288.

[2] E. Getzler, *Batalin-Vilkovisky algebras and two-dimensional topological field theories*, Comm. Math. Phys. **159** (1994), no. 2, 265–285.

[3] Yuri I. Manin, *Frobenius manifolds, quantum cohomology, and moduli spaces*, American Mathematical Society Colloquium Publications, vol. 47, American Mathematical Society, Providence, RI, 1999.

[4] L. Menichi, *Batalin-Vilkovisky algebras and cyclic cohomology of Hopf algebras*, K-Theory **32** (2004), no. 3, 231–251.

[5] ——, *Connes-Moscovici characteristic map is a Lie algebra morphism*, J. Algebra **331** (2011), no. 1, 311–337.

Larema, UMR CNRS 6093, Université d'Angers, 2 Bd Lavoisier, 49045 Angers, FRANCE

E-mail address: `firstname.lastname at univ-angers.fr`